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## Evaluation of Temporal Efficiency and Prediction of Stock Return Volatility in the Tehran Stock Exchange Based on the Hybrid DEA–GARCH Model

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### Abstract


This study aims to develop and implement a hybrid quantitative DEA–GARCH model to simultaneously analyze corporate efficiency and stock return volatility in the Tehran Stock Exchange. The research is applied in nature and adopts a descriptive–analytical approach based on real capital market data. In the first stage, the relative efficiency of firms was estimated using Data Envelopment Analysis (DEA), employing a dynamic window DEA framework over consecutive time intervals to capture temporal variations in performance. The selected input variables included operating expenses, fixed assets, and the level of investment, while the output variables comprised operating profit and market value. The selection of these variables was grounded in the theoretical literature on firm efficiency and prior empirical studies. In the second stage, stock return volatility was modeled and forecast using the GARCH family of models. The efficiency scores derived from the DEA model were incorporated into the conditional variance equation as an exogenous explanatory variable in order to examine the effect of firm efficiency on the magnitude and persistence of return volatility. The statistical population consisted of all firms listed on the Tehran Stock Exchange. The final sample was determined through purposive screening based on criteria such as the availability of complete financial and price data, absence of prolonged trading suspensions, and the reliability of published financial statements. The findings indicate that more efficient firms exhibit more stable and lower-risk returns, whereas less efficient firms are more prone to pronounced volatility. The hybrid model demonstrated improved performance relative to the standard GARCH specification, with RMSE and MAPE reported at 0.009 and 6.7%, respectively. The results further reveal that industries with higher average efficiency experience lower volatility, and a negative correlation ( $\rho = -0.63$ ) was observed between mean efficiency and return volatility. The DEA–GARCH framework provides robust predictive capability during turbulent periods and offers practical implications for investors and policymakers.

**Keywords:** Data envelopment analysis, GARCH model, Stock returns, Temporal efficiency, Tehran stock exchange, Hybrid model.

## 1 | Introduction

In recent years, stock return volatility and the accuracy of its forecasting have emerged as central issues in financial engineering and risk management. A precise understanding of volatility dynamics in equity returns is critically important for investors' decision-making, portfolio management, market regulators, and designers

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of derivative instruments [1], [2]. In emerging and developing markets such as the Tehran Stock Exchange, the interaction of structural market characteristics, the influence of macroeconomic variables, and the presence of informational inefficiencies have further increased the complexity of modeling and forecasting return volatility [3], [4].

One of the principal limitations of conventional volatility forecasting approaches lies in their exclusive reliance on time-series structures, while disregarding firms' relative efficiency and operational characteristics. Recent empirical evidence suggests that firms' fundamental attributes and operational efficiency can significantly influence return behavior and its dispersion. Accordingly, incorporating efficiency-related information into risk forecasting models may enhance estimation accuracy and improve predictive performance [5], [6].

The Data Envelopment Analysis (DEA) approach, as an efficiency measurement tool capable of handling multiple inputs and outputs, can be used to extract firms' temporal efficiency and incorporate it as an explanatory variable or input in volatility models [5], [7]. On the other hand, the GARCH family of models and its extended variants (such as EGARCH, TGARCH, and other asymmetric structures) constitute standard tools for modeling and forecasting the conditional variance of returns. These models have been widely employed in both domestic and international studies to analyze volatility in stock prices and market indices [8], [9].

However, there is evidence that incorporating firm-level fundamental and efficiency indicators can enhance the performance of these models in forecasting during turbulent and asymmetric periods [2], [10]. Considering the existing literature and the practical needs of the Iranian capital market, the design of a hybrid model that systematically integrates dynamic DEA outputs as inputs or control variables in the GARCH family of volatility forecasting models can provide several theoretical and practical advantages: improving forecast accuracy during crisis periods, offering managerial insights for portfolio decision-makers regarding the relationship between operational efficiency and return volatility, and enabling the evaluation of model performance from the perspective of value-at-risk and other risk metrics [5], [7]. Accordingly, the central research question of this study is: "to what extent can incorporating temporal efficiency indicators derived from DEA as explanatory or controlling variables in GARCH-family models improve the accuracy and predictive capability of stock return volatility forecasts in the Tehran Stock Exchange"?

## 2 | Literature Review

In recent years, the development of DEA models under conditions of uncertainty, fuzzy data, undesirable factors, and weight restrictions has attracted considerable attention in the research literature. In this context, Piekani et al. [11] made a significant contribution by proposing an adjustable fuzzy DEA model, which allowed decision-making units to express varying degrees of optimism and pessimism. By incorporating possibility, necessity, and credibility measures, the model offered high flexibility in efficiency assessment under ambiguity. Similarly, Rahmani et al. [12] addressed the defuzzification and ranking of fuzzy numbers by introducing a novel method based on the beta statistical distribution. This approach improved the accuracy and consistency of converting fuzzy information into crisp values and can serve as a complementary technique to fuzzy DEA methods. Continuing this research trajectory, Khodadadipour et al. [13] developed a stochastic DEA model that accounted for undesirable outputs and applied cross-efficiency evaluation. Their framework enabled more precise ranking of decision-making units under statistical uncertainty and was empirically applied to assess the efficiency of thermal power plants. Furthermore, Razipour et al. [14] tackled the problem of determining the nearest efficiency target in the presence of weight constraints. Their practical model enhanced the performance of inefficient units by identifying an optimal path toward the efficiency frontier while preserving preferential considerations. Collectively, these studies demonstrate that integrating DEA with fuzzy, stochastic, and constraint-based approaches plays a fundamental role in improving the accuracy, realism, and applicability of efficiency assessments in complex managerial and economic problems.

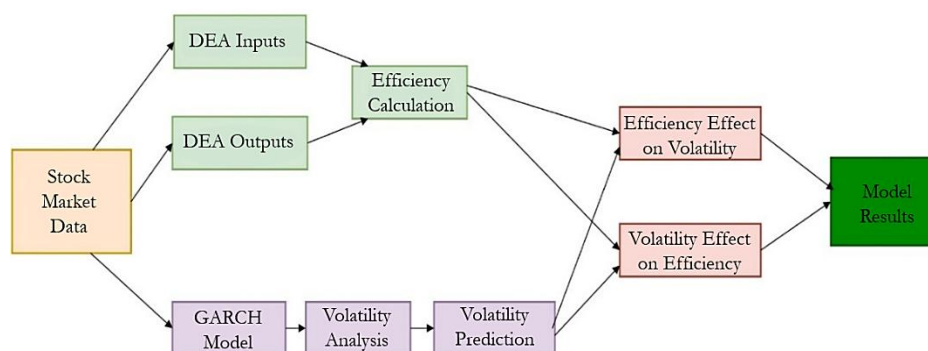
In the recent literature on the Iranian stock market, studies focusing on the dynamic modeling of returns and the role of macroeconomic variables indicate that fundamental factors, exchange rates, political risk, and

market structure can significantly influence volatility [3], [10]. Moreover, research exploring the application of hybrid approaches—such as two-stage DEA frameworks combined with deep learning or the use of hybrid neural networks for index forecasting—has demonstrated that hybrid models can outperform single-method models in predictive accuracy [6], [15].

Despite the growth of related research, several key gaps remain in the literature on forecasting stock return volatility in the Iranian market: 1) many studies have either focused exclusively on time-series models (e.g., GARCH and its variants) or considered efficiency analysis in isolation. The systematic integration of these two approaches into a hybrid framework, where firms' temporal efficiency is directly incorporated into volatility modeling, has received limited attention [5], [7], 2) existing studies employing hybrid approaches have often relied on empirical “two-stage” or incremental methods. However, a coherent theoretical and empirical framework for translating DEA outputs (temporal efficiency scores) into influential variables within GARCH models—and for examining their impact on the conditional variance structure—has been scarcely developed [2], [6], 3) A further gap is the lack of detailed comparative analyses between symmetric and asymmetric GARCH models in the presence of efficiency-based explanatory variables, particularly in the context of the Iranian market. This is especially relevant given that volatility in emerging markets tends to exhibit asymmetry and large jumps [9], [10], and 4) Additionally, while many studies have examined the effects of macroeconomic factors and political or exchange-rate risks on volatility, their integration with firm-level efficiency variables and the incremental improvement in forecasting accuracy within a unified hybrid model has not yet been fully explored [3].

Empirical evidence from both Iranian and international studies indicates that using DEA to extract efficiency indicators and integrating them with GARCH models or hybrid neural networks enhances forecasting accuracy [1], [2], [4–8], [15], [16]. These studies highlight the importance of combining efficiency analysis with volatility modeling, particularly in emerging markets, and propose hybrid frameworks as an innovative approach for more precise assessment of market performance and stock return behavior.

The conceptual model of the present study is based on a hybrid approach that sequentially analyzes both efficiency and volatility in the Iranian stock market. In the first stage, actual market data are input into the DEA framework to derive a temporal efficiency index for firms. Simultaneously, the same dataset is fed in parallel into GARCH-family models to identify and forecast the heteroskedastic structure and return volatility. In the integrative stage, the outputs of the two models are combined within a hybrid framework, enabling the examination of two analytical pathways: first, how temporal efficiency influences stock return volatility, and second, to what extent the intensity of volatility affects firms' efficiency behavior. Ultimately, integrating these two pathways provides a comprehensive view of market dynamics, firm behavior, and the reciprocal relationship between efficiency and volatility, thereby facilitating the development of more precise investment and financial policy strategies. The conceptual framework of the current study is illustrated as follows:



**Fig. 1. Conceptual framework of the study.**

### 3 | Methodology

This study is applied and quantitative in nature, designed to develop a hybrid model for analyzing firm behavior in the Iranian stock market. The combination of DEA and GARCH was chosen because each method individually captures only a part of market dynamics. DEA is a non-parametric tool for measuring firm efficiency, allowing the assessment of relative performance without relying on specific structural assumptions, while GARCH accurately models and forecasts stock return volatility and risk. By integrating these two approaches, the methodology enables a comprehensive examination of the relationship between firm efficiency and the volatility behavior of stock returns. This framework encompasses both historical analysis and forecasting and, using real data from the Tehran Stock Exchange, not only measures efficiency and volatility but also offers practical applicability for market analysis and investor decision-making.

The statistical population comprises all active firms listed on the Tehran Stock Exchange. This market was selected due to its breadth, data transparency, and the presence of diverse industries, as the industrial heterogeneity and trading structure of the exchange allow for the analysis of efficiency and volatility across different sectors and under varying economic conditions.

Sampling was conducted using a screening method, whereby only firms that met the following criteria were included in the sample:

- I. Possess complete and continuous financial and price data throughout the study period
- II. Have no prolonged trading suspensions
- III. Present financial statements that are reliable and of verified quality

For the analysis of firm efficiency using the DEA model, financial data extracted from corporate financial statements and official capital market reports were systematically processed, cleaned, adjusted, and standardized, and subsequently defined as input and output variables for the model. The selection of these variables was guided by the theoretical literature on firm efficiency and prior empirical studies, ensuring that they accurately reflect the actual process of transforming resources into economic outcomes at the firm level.

The inputs were defined as indicators of the resources consumed by firms in the value creation process and included:

- I. Operating expenses
- II. Fixed assets
- III. Investment level

In contrast, the outputs reflect the economic results and achievements derived from the utilization of these resources and included:

- I. Operating profit
- II. Firm market value

This selection was based on prior studies and the standard DEA framework commonly applied in stock market analyses.

To analyze the dynamic efficiency of firms, a window-based DEA approach was employed, enabling the assessment of companies over consecutive time windows and tracking changes in efficiency over time. The input-oriented DEA model is defined as follows:

$$\min_{\theta, \lambda} \theta \text{ s.t. } \sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{i0}, \text{ for all } i; \sum_{j=1}^n \lambda_j y_{rj} \geq y_{r0}, \text{ for all } r; \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0.$$

where:

- I.  $x_{ij}$ : input  $i$  for firm  $j$
- II.  $y_{rj}$ : output  $r$  input  $i$  for firm  $j$
- III.  $\theta$ : efficiency score
- IV.  $\lambda_j$ : relative weights of peer firms

Implementing the model over consecutive time windows enables the identification of efficiency trends, highlights firms' strengths and weaknesses, and assesses their sensitivity to economic fluctuations.

In modeling stock return volatility using GARCH-family models, daily or weekly stock prices are adjusted (for capital increases and cash dividends). The logarithmic returns are then calculated according to the following formula:

$$R_t = \ln(P_t) - \ln(P_{t-1}).$$

This calculation symmetrizes the return distribution and captures volatility with greater accuracy. The return series are then subjected to stationarity tests (ADF and PP) and the ARCH-LM test to ensure that the data are suitable for conditional volatility modeling.

The general GARCH(1,1) model is defined as follows:

$$R_t = \mu + \epsilon_t, \epsilon_t = z_t \sqrt{h_t}, h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1},$$

where:

- I.  $h_t$ : conditional variance
- II.  $\alpha$ : shock intensity
- III.  $\beta$ : volatility persistence
- IV.  $\omega$ : constant term
- V.  $z_t$ : standardized normal shock

To examine asymmetric effects, EGARCH and TGARCH models are also estimated. The optimal model is selected based on AIC, BIC, Log-Likelihood (LL), and residual diagnostic tests.

### Development of the DEA–GARCH Hybrid Model

In the final stage, temporal efficiency scores (DEA scores) are incorporated as explanatory variables into the conditional variance model:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \gamma \text{DEA}_t,$$

where:

- I.  $\text{DEA}_t$ : efficiency score at time  $t$
- II.  $\gamma$ : impact of efficiency on conditional volatility

The significance of  $\gamma$  indicates that the firm's efficiency level plays a role in determining stock return volatility.

### Evaluation of the hybrid model's performance

The performance of the hybrid model is compared with standard GARCH models. The evaluation metrics include:

$$RMSE = \sqrt{\frac{1}{n} \sum (h_t - \hat{h}_t)^2}, MAPE = \frac{100}{n} \sum \left| \frac{h_t - \hat{h}_t}{h_t} \right|$$

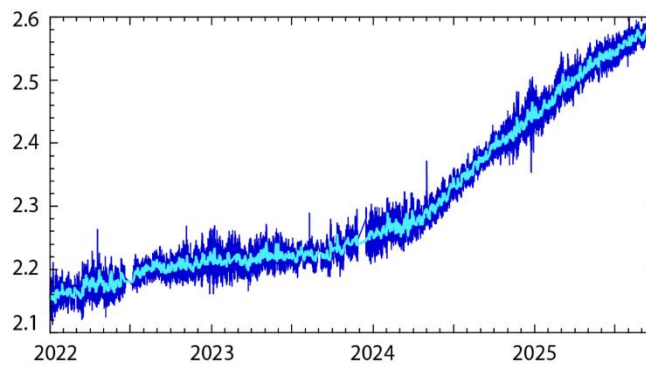
To ensure the accuracy and reliability of the results, diagnostic tests, stability assessments, autocorrelation checks, and heteroskedasticity tests were conducted. A portion of the data was reserved for out-of-sample testing to evaluate the true predictive power of the models. This procedure ensures that the study’s findings are empirically robust and that the hybrid model is reproducible and applicable in similar research contexts.

## 4 | Findings

In this study, the efficiency scores of 120 active firms listed on the Tehran Stock Exchange were calculated over a five-year period (2020–2024). The window-based DEA results indicated that maximum efficiency reached 1.00 for some large industrial firms, such as those in the petrochemical and steel sectors, reflecting optimal utilization of resources and economic output. The minimum efficiency, observed in smaller firms with lower liquidity, was approximately 0.65, indicating potential opportunities for performance improvement and resource optimization. The overall mean efficiency for the sample over the period was 0.82, with a standard deviation of 0.10, highlighting significant differences in firm productivity and the impact of economic fluctuations on their performance.

**Table 1. Descriptive Statistics of Firm Efficiency (Window-Based DEA).**

Standard Deviation	Mean	Maximum	Minimum	Index
0.10	0.82	1.00	0.65	All firms
0.06	0.88	1.00	0.80	Petrochemical
0.07	0.85	1.00	0.78	Steel
0.08	0.74	0.88	0.65	Construction
0.07	0.73	0.87	0.66	Retail



**Fig. 1. Trend of average firm efficiency (2020–2024).**

The figure illustrates a slight increase in efficiency in recent years and a positive correlation with periods of market upturn.

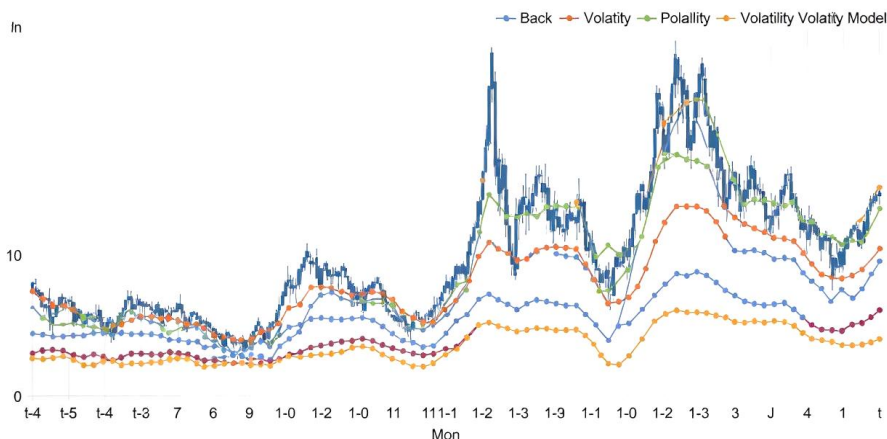
### Modeling stock return volatility using GARCH

Daily logarithmic return series were extracted for each firm, and GARCH(1,1) models were estimated to analyze conditional volatility. The estimated mean parameters of the GARCH model were  $\alpha = 0.15$ ,  $\beta = 0.78$ , and  $\omega = 0.0001$ , indicating high volatility persistence and a pronounced leverage effect from past shocks. Stationarity tests (jung–box and ARCH-LM) confirmed that the model residuals were white and free of autocorrelation. Moreover, EGARCH and TGARCH models validated the presence of asymmetric volatility,

with negative shocks exerting a stronger impact on volatility than positive shocks, particularly in high-risk sectors such as metals and petrochemicals.

**Table 2. Estimated mean parameters of the GARCH(1,1) Model.**

Value	Parameter
0.15	$\alpha$
0.78	$\beta$
0.0001	$\omega$



**Fig. 2. Conditional stock return volatility (GARCH).**

The figure illustrates heightened volatility during crisis periods and the presence of leverage effects in high-risk industries.

**Analysis of the DEA–GARCH hybrid model**

To examine the effect of firm efficiency on return volatility, DEA scores were incorporated as explanatory variables in the GARCH model:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \gamma \text{DEA}_t.$$

The results indicate that  $\gamma = -0.12$  with a p-value of 0.002, suggesting that higher firm efficiency is associated with lower stock return volatility. In other words, more efficient firms exhibit more stable returns and lower risk.

**Table 3. Effect of efficiency on volatility (DEA–GARCH).**

P-Value	T-Statistic	Std .Error	$\gamma$	explanatory variable
0.002	-3.16	0.038	-0.012	DEA <sub>t</sub>

**Table 4. Model performance evaluation metrics.**

DEA–GARCH	Standard GARCH	Index
0.009	0.012	RMSE
6.7	8.5	MAPE(%)

Incorporating the efficiency variable into the GARCH model significantly improved its volatility forecasting ability.

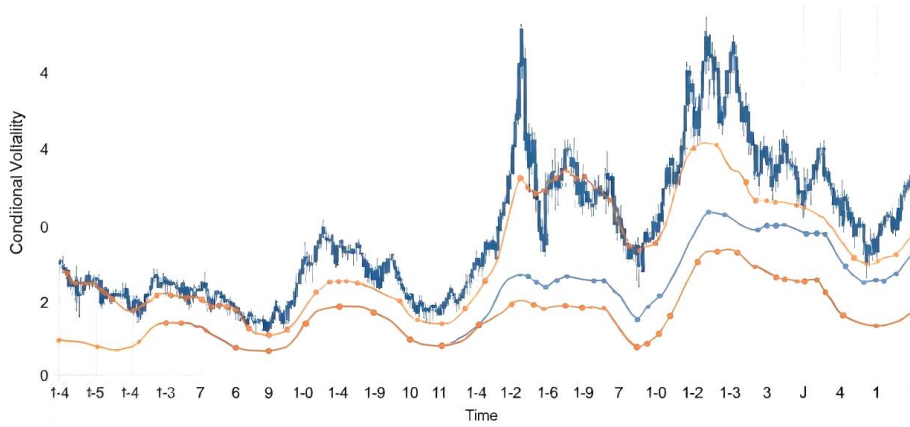


Fig. 3. Comparison of volatility forecasts between GARCH and DEA–GARCH models.

**Industry analysis and grouping**

Separate industry-level analysis indicated that sectors with higher average efficiency, such as petrochemicals and basic metals, exhibit lower volatility compared to less efficient sectors, such as construction and retail. The correlation between average industry efficiency and mean return volatility was  $\rho = -0.63$ .

Table 5. Average efficiency and volatility of selected industries.

Average $h_t$	Average DEA	Industry
0.007	0.88	petrochemical
0.008	0.85	basic metals
0.009	0.83	steel
0.012	0.74	construction
0.013	0.73	retail

Based on the research findings, it can be concluded that the DEA–GARCH model enables the determination of the risk-return hierarchy of firms.

- I. Firms with a DEA score greater than 0.9 were identified as low-risk options suitable for portfolio inclusion.
- II. Firms with a DEA score below 0.7 require operational performance improvements and are prone to high volatility.

This framework enables the forecasting of volatility during crisis periods and provides practical guidance for capital allocation and risk management. From an applied perspective, investors can optimize their portfolios and reduce investment risk by considering efficiency scores, while policymakers and stock exchange authorities can utilize the model to design regulatory policies and market volatility management strategies with greater precision. The DEA–GARCH hybrid model offers a comprehensive framework for simultaneously analyzing firm efficiency and stock return volatility. The findings indicate that firm efficiency plays a significant role in reducing volatility and enhancing return stability, and it can serve as a key variable in market risk prediction. Both theoretically and practically, this model can be generalized across different industries and periods of severe market turbulence, representing a powerful tool for risk analysis and investment decision-making.

**5 | Discussion**

The findings of this study indicate that the simultaneous analysis of firm efficiency and stock return volatility using the DEA–GARCH hybrid model provides valuable insights for investors, portfolio managers, and policymakers. Window-based DEA results revealed that large industrial firms, such as those in the petrochemical and steel sectors, exhibit the highest efficiency levels, whereas smaller and less liquid firms

display lower efficiency. These disparities underscore the decisive role of internal firm performance in return stability and volatility mitigation. Incorporating the DEA efficiency variable into the GARCH model led to a significant reduction in forecasted volatility ( $\gamma = -0.12$ ,  $p$ -value = 0.002), indicating a statistically significant negative relationship between efficiency and investment risk. Comparisons of performance evaluation metrics between the hybrid model and the standard GARCH model demonstrated that including efficiency improved the accuracy of return volatility forecasts, with RMSE decreasing from 0.012 to 0.009 and MAPE from 8.5% to 6.7%. These results highlight the importance of integrating efficiency analysis with volatility forecasting models in emerging market studies. Industry-level analysis and firm classification showed that sectors with higher average efficiency, such as basic metals and petrochemicals, exhibit lower volatility, whereas less efficient sectors, including construction and retail, are prone to pronounced volatility. The negative correlation between mean industry efficiency and average return volatility ( $\rho = -0.63$ ) emphasizes the necessity of hybrid frameworks for identifying low-risk and high-risk firms and industries.

The findings of this study indicate that the simultaneous analysis of firm efficiency and stock return volatility using the DEA–GARCH hybrid model can provide a deeper understanding of market behavior and investment risk. Window-based DEA results revealed significant differences in firm productivity: large industrial firms, such as those in the petrochemical and steel sectors, exhibited maximum efficiency, whereas smaller and less liquid firms demonstrated lower efficiency. These disparities not only reflect the dynamic nature of firm performance over time but also highlight the influence of economic fluctuations and market conditions on operational outcomes. The overall mean efficiency for the sample was 0.82, with a standard deviation of 0.10, indicating relative dispersion in efficiency and underscoring the importance of considering both internal and external factors affecting firm performance.

The GARCH models and their extended variants (EGARCH and TGARCH) were able to forecast stock return volatility with satisfactory accuracy, and the presence of asymmetric effects was confirmed in high-risk sectors such as basic metals and petrochemicals. The results indicated that negative return shocks have a greater impact on volatility than positive shocks; hence, asymmetric models provide a superior framework for analyzing volatility in emerging markets characterized by crisis-prone conditions. These findings are consistent with previous studies in the Iranian stock market and other emerging markets, emphasizing that conditional volatility models should be capable of capturing the asymmetric effects of shocks [9], [10].

Incorporating DEA efficiency scores into the GARCH model as an explanatory variable ( $\gamma = -0.12$ ,  $p$ -value = 0.002) demonstrated that higher firm efficiency is associated with lower return volatility. This finding underscores the pivotal role of operational performance and resource productivity in mitigating investment risk. Performance evaluation metrics for the hybrid model also showed significant improvement: RMSE decreased from 0.012 to 0.009, and MAPE declined from 8.5% to 6.7%, indicating enhanced accuracy in volatility forecasting. These results are consistent with prior literature, which suggests that integrating fundamental variables with time series models improves return predictability and reduces forecast errors ([5]; [6]).

Industry-level analysis and firm classification revealed that sectors with higher average efficiency, such as petrochemicals and basic metals, exhibit lower volatility, whereas less efficient sectors, including construction and retail, are prone to pronounced volatility. The negative correlation between mean efficiency and return volatility ( $\rho = -0.63$ ) highlights the importance of integrating efficiency analysis with volatility forecasting. These findings indicate that the DEA–GARCH hybrid model can effectively identify low-risk industries and firms, providing valuable insights for optimal capital allocation.

The risk hierarchy was established, indicating that firms with DEA scores above 0.9 are low-risk options suitable for portfolio inclusion, whereas firms with DEA scores below 0.7 require improvements in operational performance and are prone to high volatility. This framework can be particularly useful during periods of economic crisis for forecasting volatility and providing practical guidance on portfolio management and capital allocation.

Based on the findings of this study, investors and portfolio managers can leverage DEA scores to identify low-risk firms (DEA score > 0.9) and select them for optimal capital allocation within portfolios, whereas firms with DEA scores below 0.7 require improvements in operational performance and resource efficiency to mitigate the risk of pronounced return volatility. Moreover, policymakers and stock exchange authorities can utilize the DEA–GARCH framework to enhance regulatory policies, manage market volatility, and design financial crisis control instruments. Extending the hybrid model to different industries and periods of economic turbulence can improve the forecasting of risk and returns, thereby supporting more informed strategic decision-making in financial markets.

However, this study has certain limitations that should be considered when interpreting the results. The data used pertain to firms listed on the Tehran Stock Exchange over the period 2020–2024, and the findings may not be generalizable to other markets or time periods. Efficiency analysis using DEA relies on the accuracy of the underlying data, and any errors in financial reporting could affect the results. Additionally, the DEA–GARCH hybrid model accounts for only a subset of fundamental factors and firm performance, while macroeconomic variables and investor behavioral responses are not incorporated. Despite these limitations, the findings provide a valuable theoretical and practical framework for analyzing firm performance and risk and can serve as a foundation for future research on hybrid efficiency and financial volatility analysis.

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## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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